Parametric versus non-parametric simulation

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Abstract

Most of ex-ante impact assessment policy models have been based on a parametric approach. We develop a novel non-parametric approach, called Inverse DEA. We use non parametric efficiency analysis for determining the farm's technology and behaviour. Then, we compare the parametric approach and the Inverse DEA models to a known data generating process. We use a bio-economic model as a data generating process reflecting a real world situation where often non-linear relationships exist. Results suggest that traditional parametric approaches are biased and inconsistent. The Inverse DEA model under variable return to scale preserving technical efficiency scores outperforms any other specifications. However such non-parametric approach is by nature sensitive to noise which hampers its accuracy when it prevails. The use of panel data is preferable.

Keywords: *parametric; Inverse DEA; simulation; policy model*

1. Introduction

Ex-ante impact assessment policy models aim to understand and model the evolving dynamics behind farmer behaviour. Typically, the only information available to researchers is an input and output dataset expressed in monetary value or physical units with sometimes additional price information. The estimated policy models aim to describe the process that produces the data in an observed population, also called the Data Generating Process (DGP). The key question is what can we learn from this limited information? In other words, how can we use at best the information in a dataset to estimate relationships between inputs and outputs and so to mimic the DGP? Before being able to answer these questions, it is essential to be aware of the properties of whatever estimator used, requiring us to evaluate the consistency of the estimators. As researchers are never in the position to know the 'true' DGP often reasonable assumptions are made. These assumptions enable the estimation using estimators that have desirable properties and so allowing these properties to be inferred. At the same time, these assumptions should not impose any restriction that does not reflect reality which could lead to misleading conclusions. Indeed, any conclusion is valid to the extent that assumptions made are true. The different estimation techniques are like a continuum ranging from fully parametric estimation, passing by semi-parametric estimation to fully non parametric estimation. This goes in pair with the number and the strength of assumptions made. In this paper we compare two opposite estimation approaches: a parametric estimation approach where a production function is specified and a non-parametric approach where some assumptions are made only on the sample distribution. We test the aptitude of both methodologies to reveal a known bio-economic DGP.

The choice of the DGP plays a determinant role. First of all, a DGP similar to one of the chosen estimation techniques will certainly favour it. Hence, a bio-economic model has been chosen as it is conceptually very different from both estimation approaches. Second, in the agricultural sector binding constraints are common, such as feed and fertilization requirements. Hence, any ex-ante policy models should be able to reveal these underlying discontinued relationships.

Most of the agricultural policy models have been based on a parametric approach (see Heckelei et al, 2012), involving the specification of a production function, often a variable cost function taking typically the quadratic form. The main disadvantage of such method lies in the bias in the case of a misspecification of the functional form is miss-specified. Despite the fact that farmers rely on biological process subject to inequality constraints, it is often assumed that a parametric approach is a good approximation of the reality. Knowing that non linearity exists, we choose to evaluate a very flexible form of the production function while imposing economic properties such as monotonicity and concavity.

On the other hand, we test a novel non-parametric approach enabling us to ex-ante predicts the impact of policy changes. The concept of this approach is to use non parametric efficiency analysis as a methodology for defining the farmer's technology and behaviour. The simulation model maximises revenue and uses technical efficiency scores of each individual and the frontier as specification of the technology. It avoids specifying a functional relationship between inputs and outputs. Contrary to the parametric approach, the production function is determined by the data itself and the effect of the explanatory variables. Hence the arbitrary choice of a specified production function is removed preventing the misspecification of the distribution of inefficiency terms.

The objective of this paper is to compare a parametric simulation versus a non-parametric simulation and to evaluate them against a known DGP. The particularity here is that a bio-economic model is used as the DGP. For both approach we test whether or not estimator are consistent and whether or not they are biased. Finally we look to which extend inferences can be drawn. The paper is organised as follow: Section 2 describes the bio-economic DGP, the different samples that we generates from our DGP and the different performance criterion used in this study. Section 3 explains the parametric approach adopted in this paper. Section 4 illustrate the non-parametric methodology and provides an explanatory example and section 5 provides results of our simulation exercise. Finally Section 6 concludes, discusses limitations and further possible researches.

2. Data Generating Process

To evaluate the performance of the methods, many authors have used a parametric specification for the data generating process (DGP) (see Krüger, 2012, Andor and Hesse, 2011 and, Banker et al., 1993). The disadvantage of using either a parametric approach or a non-parametric efficiency approach to compare both is that the results are predictable and do not provide insights about the reliability of the methods when the DGP is not known. A parametric specification of the DGP may certainly influence the results in favour of the parametric specification using the same functional form.

A better and more extreme test of the methods is to feed them with a DGP that is conceptually very different both from the parametric and non-parametric policy models. Therefore, our approach uses a bio economic DGP. Furthermore, such DGP reflects the reality of the agricultural sector where often resource endowments are binding constraints in the short run.

2.1. Specification of the DGP

To do so, we choose to simulate the optimal feed input ration of a dairy cow. To ease understanding of the model only three different feed inputs can be chosen and only one output is simulated namely milk production. The objective of the model is to maximise profit given different input prices and output prices subject to feeding constraints. This is computed using General Algebraic Modeling System (GAMS) software and as our model is a non-linear optimisation problem we use the CONOPT solver.

For a given sample of n observation with each observation j $(j = 1,\dots,n)$ we can generate our data with the following non-linear optimisation problem (1):

$$Max_{X_{k,j},Y_{r,j}} \sum_j \left[Y_{r,j} \times p_{r,j} - \sum_k (X_{k,j} \times p_{k,j}) \right] \tag{1}$$

S.t.

$$\sum_k X_{k,j} \times Intake_{capacity_{k,j}} \leq (\varepsilon_j + Max_{Intake_{capacity_j}}) + Conversion\_Factor_{capacity} \times Y_{r,j}^2$$

$$\sum_k X_{k,j} \times Intake_{energy_{k,j}} \geq Min_{Intake_{energy}} + Conversion\_Factor_{enery} \times Y_{r,j}^2$$

$$\sum_k X_{k,j} \times Intake_{protein_{k,j}} \geq Min_{Intake_{protein}} + Conversion\_Factor_{protein} \times Y_{r,j}^2$$

Where profit is maximised given the fact that each observation faces different input prices $p_{k,j}$, and output prices $p_{r,j}$. Prices are drawn from a uniform distribution of (0.5, 1.5). The first constraint ensures that the total feed intake does not exceed the maximum feed intake capacity of the cow. An error component ε_j is endogenously added and can have the following distribution (2):

$$\varepsilon_j \sim N(0;0) or\ \varepsilon_j \sim N(0;0{,}05)\ or\ \varepsilon_j \sim N(0;0{,}2)\ or\ \varepsilon_j \sim N(0;0{,}5) \tag{2}$$

In other words, we introduce different coefficients of variation to the maximum feed intake capacity of 0, 2.3, 9.1 and 22.7. When the coefficient of variation is 0, each observation is defined by an identic relationship between inputs and output. The dataset generated can be considered as a time series where the feed intake of one dairy cow is recorded for different prices that could have been observed at different point in time. As soon as an error term is introduced on the right hand side of the first constraint, the maximum feed intake capacity is different for each observation and we can consider that we generate a cross sectional dataset of several dairy cows at one point in time facing different prices. The second and the third constraints ensure respectively that the energy and protein requirements are full fill. Furthermore we investigate the effect of the sample size, we generate six different sample sizes: 1000, 500, 200, 100, 50 and 10 observations.

So, the information that is made available to estimate the parametric and the non-parametric models is a sample[1] (3)

$$\begin{aligned} &X_{k,j} + \varepsilon_{k,j}\ with\ (k = 1,2,3) \\ &Y_{r,j} with\ (r = 1) \end{aligned} \tag{3}$$

of *n* observation, with each observation *j (j = 1,…,n)* using three inputs and generating one output. However in reality, researchers often assume statistical errors which might have difference source of origins such as measurement errors made during the collection of data. For this reason, we also add an exogenous error term with the same distribution than mentioned in (2). In total we generate 96 different samples.

Then, we need points of reference against which the accuracy of both approaches will be tested. In the context of ex-ante agricultural policy models, we intend to predict for instance the change of inputs used due to price changes. Therefore, we implement price changes of inputs by running model (1)[2] holding output quantities and output prices fixed. We obtain the following set of inputs:

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[1] In order to simplify the notation, sample generated from our DGP are in capital letter (X_j, Y_j) while the estimates are in lowercase letter (x_j, y_j).

[2] Problem (1) is a non-linear problem where we use the CONOPT solver. It has to be note that CONOPT does not guarantee optimum solution but rather a feasible solution. Additionally, for efficiency reasons it is often recommended to specify initial starting values of the variables. In order to not bias result and be as close as possible of real situations, we use the set of initial inputs $X_{k,j}$ as initial starting values.

$X^*_{k,j}\ with\ (k = 1,2,3)$ (4)

Where $X^*_{k,j}$ is the new set of inputs optimising profit under the new price conditions and leading to the same amount of output than previously. It has to be noted than the new prices are drawn from the same distribution than the initial prices. The question is to which extend the estimated parametric and non-parametric models will be able to simulate the same set of inputs $X^*_{k,j}$ after price changes.

2.2. Simulation Scenario

The ultimate goal here is to measure the difference between the known DGP and the other scenarios in term of changes in input quantities due to price changes. The models tested are the parametric model named "Restricted Linear Regression model" (RLR) versus the non-parametric Inverse DEA model. We refer to the parametric approach as a restricted model as we impose some important economic properties such as monotonicity and concavity. The inverse DEA model can either be specified under Constant Return to Scale (CRS) or under Variable Return to Scale (VRS). Figure 2 provides an overview of the comparison procedure:

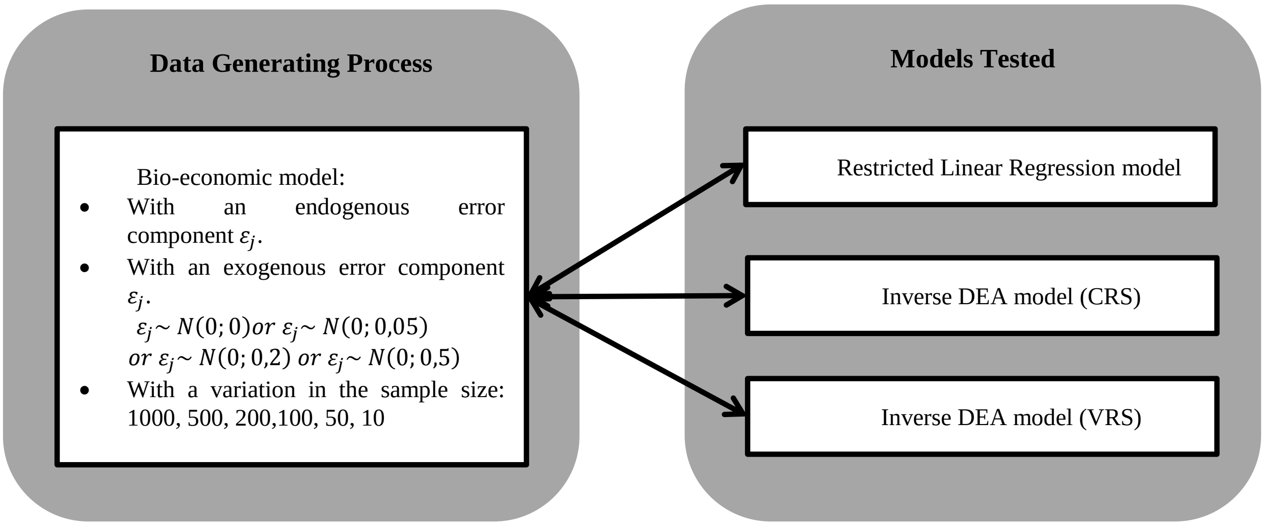

Figure 1. Simulation design

2.3. Performance criterion

The accuracy of the simulation models can be defined by to the bias and efficiency. T Different types of statistical bias exist: measurement, sampling, and estimation (Walther et al., 2005). As their name implies, measurement and sampling bias refer to the intrinsic structure of the data set and usually do not disappear with an increase of the sampling effort. In our simulation setting measurement bias is only captured by the introduction of an exogenous error term in the DGP. Sampling bias is not introduced and thus not considered in this paper. Estimation bias is the difference between the estimated input changes and the 'true' input changes due to prices change. Estimation bias should decrease with an increasing sample size. Although the model could be unbiased, it is important to take into account the variance. Indeed, if the model is biased, an increase of the sample size just decreases the variance around a wrong estimate. On the other hand, if both the variance and bias tend towards zero, then our model is consistent.

Bias measure take typically into account the difference between the estimated value and the true value. One common measure is the Mean of the Mean Deviation (MMD) for each observation which corresponds to the difference between the new quantities of the input mix after price changes from the DGP noted $X_{k,j}^{*}$, and the predicted quantities of the input mix after price changes calculated with the different scenarios noted $x_{k,j}$. Another indicator is the Mean of the Mean Absolute Deviation for each observation (MMAD). It takes into account the difference between the estimated input changes and the true input changes, but also eliminates the direction of the difference taking into account the variance of the estimates. The MMAD assesses the overall consistency of the model taking into account bias and efficiency. The mathematical formula of the MMD and MMAD are the following (5):

$$MMD = \frac{\sum_{j=1}^{n}(X_{k,j}^{*} - x_{k,j})}{n} \tag{5}$$

$$MMAD = \frac{\sum_{j=1}^{n}\left|X_{k,j}^{*} - x_{k,j}\right|}{n}$$

To gain additional insight, we also used graphical descriptive statics such as boxplot of the mean deviation (MD) as a performance criterion.

3. The parametric approach

As parametric approach, we test a very flexible functional form as it promises a better fit to the data. We tested several polynomial functions until the fourth order, however for parsimony reasons we only present the results of the most relevant one naming cubic production function and dropped when necessary the non-significant parameters. Unfortunately, a third order flexible functional form comes at the cost of economic properties such as monotonicity and curvature being violated. Therefore, we impose concavity and monotonicity for each observation permitting to choose inputs level that maximize profit.

We use Ordinary Least Square methodology to estimates the unknown parameters. Let's consider a set of *n* observation, with each observation *j (j = 1,…,n)* using *i* inputs $X_{k,j}$*(k=1,…, i)* and generating *s* outputs $Y_{r,j}$*(r=1,…, s)* obtain from problem (1). We can derive the following non-linear optimisation problem (6):

$$Min_{\beta_k,\delta_{k,l},\gamma_{k,l,m}} \sum_{j} \varepsilon_j^2 \tag{6}$$

S.t

$$Y_{r,j} = \alpha + \sum_{k} \beta_k (X_{k,j} + \varepsilon_j) + \sum_{k}\sum_{l} \delta_{k,l}\,(X_{k,j} + \varepsilon_j)(X_{l,j} + \varepsilon_j) + \sum_{k}\sum_{l}\sum_{m} \gamma_{k,l,m}(X_{k,j} + \varepsilon_j)(X_{l,j} + \varepsilon_j)(X_{m,j} + \varepsilon_j) \quad + \varepsilon_j$$

$$\frac{\partial Y_{r,j}}{\partial (X_{k,j} + \varepsilon_j)} \geq 0 \text{ for } k, l, m = l, \dots, i$$

$$\frac{\partial^2 Y_{r,j}}{\partial^2 (X_{k,j} + \varepsilon_j)} \leq 0 \text{ for } k, l, m = l, \dots, i$$

where $\beta, \delta \; and \; \gamma$ are the parameters to be estimated. According to economic theory *Y* must be monotonic and concave. The first and second constraints ensure monotonicity and concavity respectively.

Once we have estimated the production function we can simulate changes in input mix due to a change in input prices while keeping output fixed. We then solve the following non-linear optimisation model (7):

$$Max_{y_{r,j}x_{k,j}} \sum_j \left[Y_{r,j} \times p_{r,j} - \sum_k (x_{k,j} \times p^*_{k,j}) \right] \tag{6}$$

S.t

$$Y_{r,j} = \alpha + \sum_k \beta_k x_{k,j} + \sum_k \sum_l \delta_{kl}\, x_{k,j} x_{l,j} + \sum_k \sum_l \sum_m \gamma_{klm} x_{k,j}\, x_{l,j} x_{m,j} + \varepsilon_j$$

$$\frac{\partial Y_{r,j}}{\partial x_{k,j}} \geq 0 \; for\; k,l,m = l,\dots,i$$

$$\frac{\partial^2 Y_{r,j}}{\partial^2 x_{k,j}} \leq 0 \text{ for } k,l,m = l,\dots,i$$

where we maximise profit after change in input prices $p^*_{k,j}$ subject to the production function constraint estimated in problem (5), monotonicity and concavity constraints.

4. The Inverse DEA approach

Data Envelopment Analysis (DEA) is used here as a methodology which approximates the production function by a piece-wise linear envelopment supported by the observed data points, that contain all data points and that respect some economics properties such as monotonicity, convexity and return of scale. This concept has already been applied by Speelman et al. (2009), Frija et al. (2011) and Oude Lansink et al. (2008). However, to the best of our knowledge none papers have assessed the accuracy of such models.

Beyond efficiency measurement, Wei et al. (2000) extended the inverse optimization problem to the DEA framework for short-term input and output estimation. Frija et al. (2011) and Speelman and al. (2009) have applied such Inverse DEA model in the field of forecasting and resource allocation. In this configuration inputs and outputs are used as parameters to determine objective values, in other words their efficiency scores. The model can estimate the needed changes in the input combination due to policy change while preserving the initial technology frontier. The model is input oriented and can exhibit constant returns to scale (CRS) or variable return to scale (VRS). Input substitution is feasible given the change in policies. The procedure is in two steps. First the firm-level inefficiency is measured with DEA and next the observed peers are used as piecewise linear technology frontier during simulation.

4.1. Measuring firm-level inefficiency with DEA

DEA uses linear programming methods to construct a non-parametric piecewise frontier which envelopes the observed input and output data for all observations. In this paper we used the models proposed by Charnes, Cooper and Rhodes, (1978) and Banker, Charnes and Cooper, (1984). For a given observation denote with the subscript *o*, we can derive the following linear problem, known as the *envelopment form* (7) :

$$Min_{\theta,\lambda_j}\, \theta_o \tag{7}$$

S.t.

$$\sum_{j=1}^{n} \lambda_j Y_{r,j} - Y_{r,o} \geq 0,$$

$$\theta_o X_{k,o} - \sum_j^n \lambda_j X_{k,j} \geq 0, \qquad with\ (k = 1,2,3)$$

$$\lambda_j \geq 0$$

$$\sum_{j=1}^{n} \lambda_j = 1, under\ VRS$$

where θ is a scalar and λ is a $n \times 1$ vector of constants. The value of θ obtained is the technical efficiency (TE) score of the *i*-th firm. This model is referred to as providing a *radial projection* of inefficient observations on the frontier. Specifically, each input is reduced by an equi-proportional factor θ.The value of θ lies between zero and one, with a value of one indicating that considered observation is on the frontier and so technically efficient. The problem (7) must be solved *n* times, once for each observation.

4.2. Inverse DEA simulation

The second step is to apply an Inverse DEA model to simulate the impact of policy changes, an input price change $p^*_{k,j}$. Here, we need to calculate the new optimal input level of for a given observation noted $x_{i,o}$ which maximises revenue while at the same time preserving the TE score of DMU*o* found in problem (1). We can derive the following linear problem (8):

$$\max_{\lambda_{jpeer}, xsim_{ce}} \sum_{r=1}^{s} p_{r,o} Y_{r,o} - \sum_{k=1}^{m} p^*_{k,o} x_{k,o} \; with \; (k = 1,2,3) \tag{8}$$

S.t.

$$\sum_{jpeer}^{n} \lambda_{jpeer} Y_{r,jpeer} \geq Y_{r,o}$$

$$\sum_{jpeer}^{n} \lambda_{jpeer} \, x_{k,jpeer} \leq \theta_o xsim^*_{i,o}$$

$$\lambda_{jpeer} \geq 0$$

$$\sum_{j=1}^{n} \lambda_{jpeer} = 1, under \; VRS$$

The model maximizes the profit of observation *o* given its output price $p_{r,o}$ and its new input prices $p^*_{k,o}$. The constraints make sure that the efficiency score θ_o remains the same and *jpeer* refers to the peer observations defining the production frontier. The new optimal input vector $x_{k,o}$ calculated by the model leads to a new observations maximising profit and preserving initial efficiency score.

We now illustrate our model and its solution method with a simple example under CRS. Let's consider three observations with two inputs and one output. The data of inputs and output are shown in the following table 1.

Table 1: Inputs and output of a simple example

	A	B	C
Input 1	2	4	3
Input 2	4	4	2
Output	1	1	1
TE scores	1	0.66	1

The model is illustrated in Figure 2, B is technically inefficient, and its production frontier is P(B). Its technical inefficiency score derived from model (1) is 0.66. For the respective input one and two, the market prices are one and three represented by the initial isocost line in Figure 1. A and C are technically efficient but only C is cost efficient, meaning that its inputs combination minimises cost.

Now we assume that the prices of both inputs change, and that the slope of the new iso-cost line is -4. As a result B will shift along its production frontier P(B) in order to maintain its technical efficiency score, in the meantime, the model tries to find a point that minimise cost. If we imagine a parallel shift of the new isocost line until it becomes tangent to P(B), the tangency point Bn is the input combination preserving inefficiency of B and minimising its costs under the new price conditions. A remains the same as its inputs combination minimise costs and C adopts the same input combinations that A.

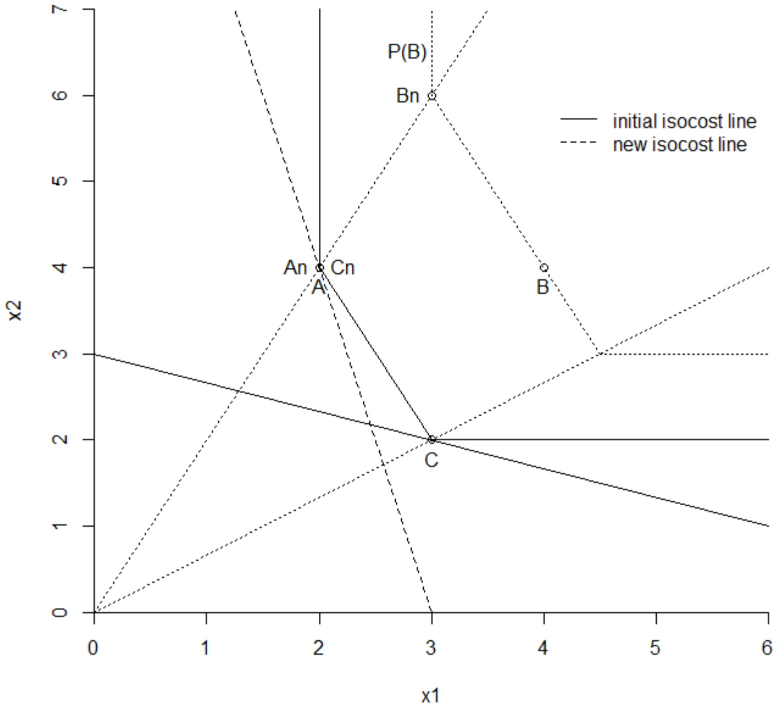

Figure 2: Simulating effect of price changes in a simple example

5. Results

Let's first examine the MMAD and the MMD of the different models for six sample sizes and without any error terms added to the DGP whether endogenously or exogenously. The solid lines in Figure 3 shows the changes of the MMAD scores with changing sample size while the dashed line illustrates the impact of the sample size on the MMD scores.

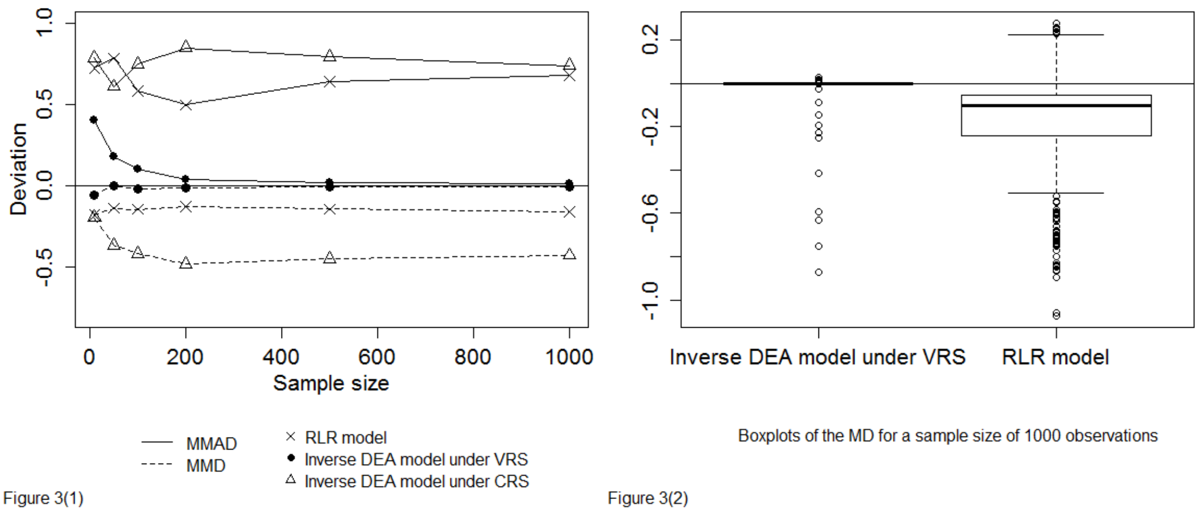

Figure 3. MMAD and MMD of the different tested model from a DGP without any error component

In such faultless setting, we can consider that our DGP generates a time series dataset of one dairy cow for different prices free of measurement bias (see 2.1). The Inverse DEA model under VRS outperforms any other model independently of the sample size variation. Its MMAD decreases towards zero with an increase of the sample size, indicating the consistency of the model. Hence, the DEA model under VRS is consistent and so able to reveal the underlying DGP as the sample size increases infinitely. The second criterion, MMD, provides information about over- and under-estimation, which is bias of the models. For a sample size of 10 observations, the Inverse DEA model under VRS performs very well although a small negative MMD prevails. The model tends to slightly overestimate input changes but bias decreases with an increase of the sample size. Figure 3(2) shows boxplots of the input change estimates from the Inverse DEA model under VRS and the RLR model. Recalling that the efficiency is measured by the variance of our estimates, the boxplots of Figure 3 indicates that the Inverse DEA model performs in general very well as the variance is very low, but that some outliers may hamper results.

Regarding Inverse DEA under CRS, its MMAD is large and more importantly it is not affected by an increase of the sample size. The Inverse DEA model under CRS is inconsistent and fails to capture the true DGP. In reality, our DGP, being a bio-economic model, is constrained by maximum feed intake capacity. Under CRS, several inputs can increase proportionally and indefinitely as long they maximise profit. Figure 3(1) shows that the MMD of this model is the largest of the three estimation methods. The CRS model overestimates input changes and is therefore biased and inconsistent. Under VRS some inputs cannot increase above the Non-Increasing Return to scale facet of the production frontier. In fact, the production frontier under VRS envelops more tightly the data and corresponds therefore better to the true DGP.

Similar conclusions for the RLR can be drawn than for the Inverse DEA model under CRS. We do not observe any conclusive trend of the MMAD with an increase of the sample size. This means that the estimators for the inverse DEA under CRS and the RLR models are inconsistent. As mentioned earlier, our DGP is based on a linear objective function subject to inequality constraints. Consequently, explanatory variables are not continuously differentiable, meaning that for instance a change in feed inputs leads to a discontinuous change in input cost. Then, any parametric specifications of the production function are by design incorrect. Nevertheless it is generally assumed that a flexible specification of the production function such as the cubic specification is a good approximation. Here, we illustrate that even a flexible production function such as the cubic function, is not able to reveal the underlying DGP despite the fact that concavity and monotonicity have been imposed. Figure 3(1) confirms these findings showing that first the MMD is negative and so the model is biased. Second the variance is large showing the lack of efficiency and so the inconsistency of the model.

Though Inverse DEA model under VRS seems to outperform any other estimation methods, such non parametric methods rely heavily on the dataset's quality and are known to be sensitive to noise or outliers. In the second step of the assessment of the model an endogenous error term is introduced in the specification of our DGP, allowing us to generate cross sectional data. To which extend individual observations differ of technologies depend on how large is the variance of the error component. We also test the impact of the introduction of an error term exogenously because also in reality the dataset may contain some measurement errors. The Figure 4 and 5 show on the first column the performance of the different estimation technics against the known DGP when an endogenous error term is included and progressively increases. The second column of Figure 4 and 5 shows the performance of the different models when an exogenous error term is included and progressively increases. In both situations the introduction of one error term excludes the other.

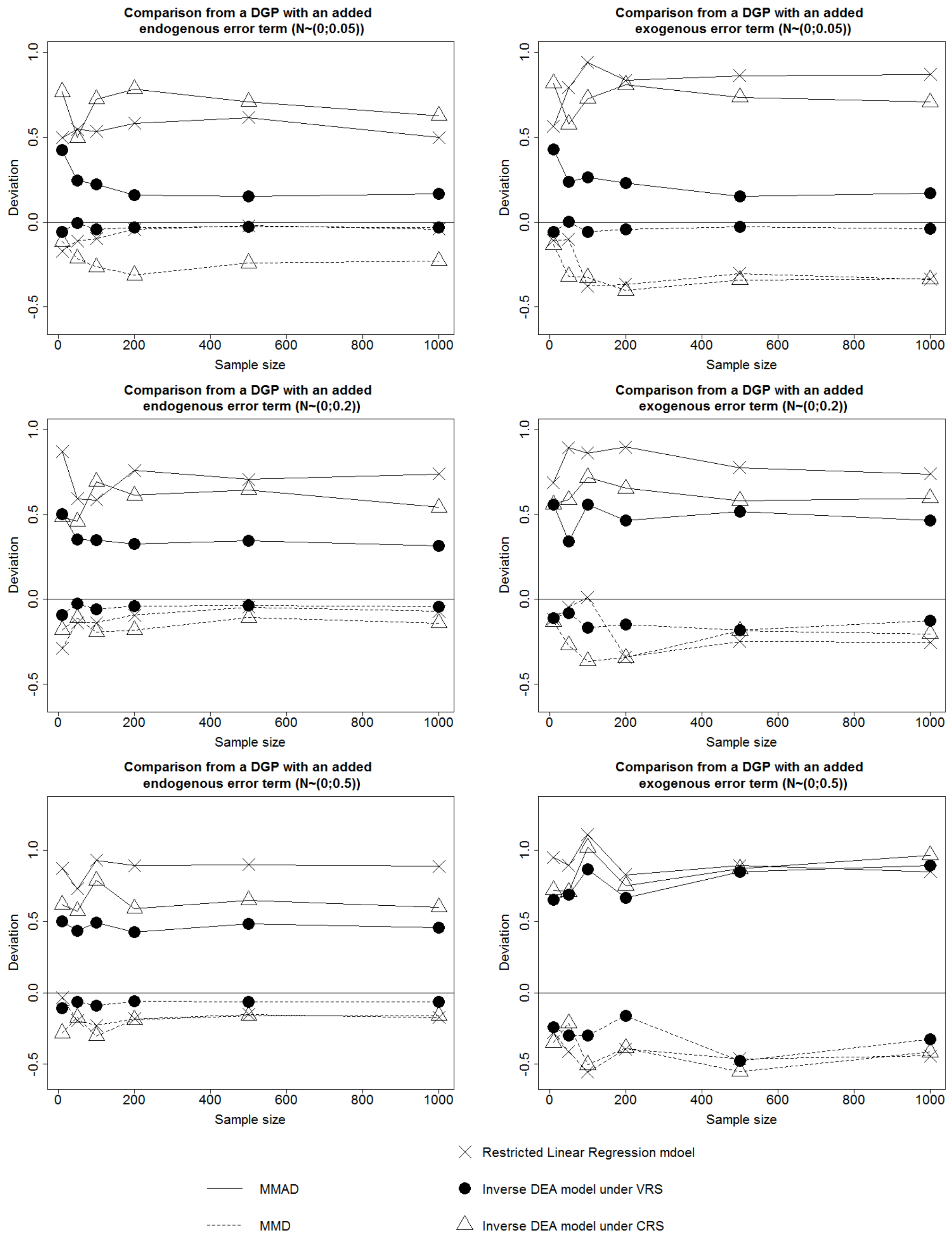

Figure 4. MMAD and MMD of the different tested model from a DGP including an error component

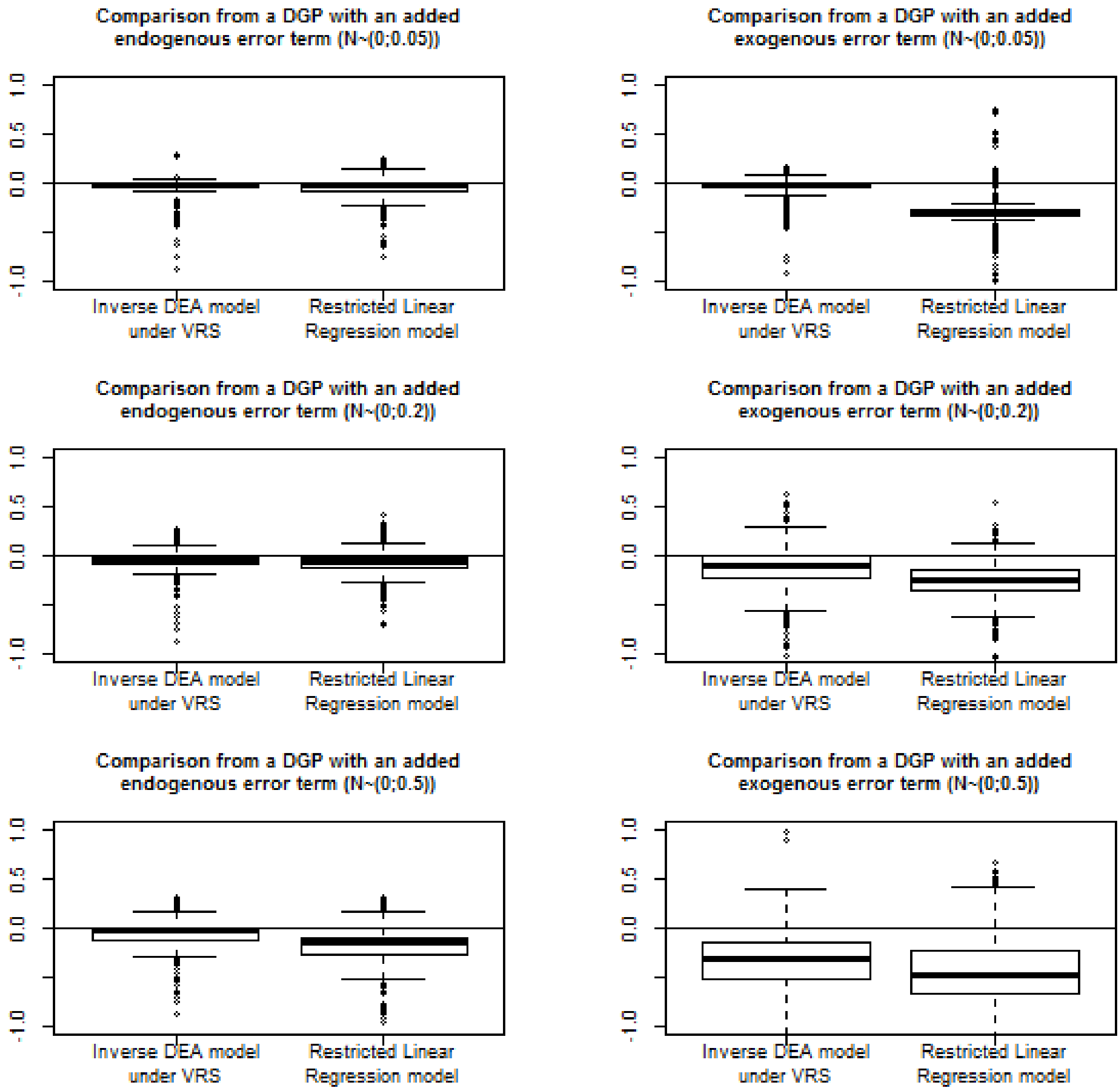

Figure 5. Variance of the MD of the different tested models from a DGP including an error component

Figure **4** shows that the Inverse DEA model under VRS performs the best relative to the other, independently on how large is the variance of the error term. Yet, the MMAD of the Inverse DEA model tends to converge toward a value meaning that the model is consistent. However, its MMAD converge at a lower speed and the MMD becomes larger as the variance of the endogenous error term increases. The model is thus upward biased and becomes less efficient with increasing errors. Figure **5** shows that the variance increases along with an increase of the variance of the error term confirming that the Inverse DEA model under VRS is consistent, biased upwards and that its efficiency depends on the extend the technology of each observations differs from each other. Indeed, by introducing an error term with in the feed intake capacity constraint introduces variation in the stomach capacity. Therefore, we do not simulate the DGP of one cow but of different cows indicating that it rather mimics the situation of cross-sectional data. With a small variance of the error term the stomach capacity of cows are still very similar.

In the situation where we introduce an exogenous error term, meaning that our DGP generates panel data and we introduce measurement bias, the Inverse DEA model under VRS performs reasonably good only for the smallest variance of the error term. The right column of Figure **4** shows that the MMAD on the Inverse DEA model under VRS is the smallest compare to other models. However, when the variance of the error term increases the MMD becomes more and more negative and the MMAD does not seems to converge to any value. Furthermore, Figure **5** indicates that the variance of MD increases drastically with the largest error term. In such setting the model is biased upward, inefficient and inconsistent. This represents an expected result. Indeed, the Inverse DEA model is a non-parametric deterministic model, and by nature fail to separate noise from the estimators. When noise prevails, the model is inaccurate and leads to inconsistent estimations.

The RLR model applied on the DGP including an endogenous error term has a negativeand a large and increase variance with an increasing error term in the DGP. Moreover, the MMAD does not decrease along with an increase of the sample size, indicating that the model is upward biased, inefficient and inconsistent. Comparable conclusion can be drawn for the RLR model applied on a DGP including an exogenous error term. These results seem to go against the common assumption that any analytic function converges to a Taylor polynomial approximation. However, there is no real contradiction because an analytic function is one that is infinitely differentiable. The bio-economic DGP is not an analytic function and can therefore also not be approximated by a flexible functional form as the third order Taylor polynomial. Actually, no analytic function exists of the bio-economic DGP and therefore the RLR model is biased and inconsistent.

Overall, the Inverse DEA model under VRS performs better, but such models are very sensitive to the quality of the dataset and do not support significant noise. The RLR models are not able to capture the true behaviour of the DGP as by design they cannot account for binding constraints.

In this experiment we chose a DGP that we run for a different set of prices in order to generate our production frontier. We saw that in the case where the DGP generates a cross sectional dataset, Inverse DEA model under VRS perform well only if the technologies among observations do not differ too much. We can wonder if it is feasible to use such type of methodologies on a dataset composed of individuals having their own DGP, for instance, a group of farmers having the same activities in one region. Despite that the group could be relatively homogeneous, each farmer has its own behaviour and rational. In other words, can we use cross sectional data to predict change of a single observation and assume that this observation will shift toward the particular “DGP” of another one? In this case, the use of grouping technics in order to identify observation with similar technologies and the use resampling techniques to correct for bias could be potential remedies.

6. Conclusion

One of the main challenges in the practice of ex-ante simulation is to construct the most accurate model that captures the underlying behavior of the DGP. Often researchers choose to parametrically estimate a functional form. By adopting a functional form we often assume a continuous relationship among variables. The continuity of these relationships is rarely happening in the real world situation, but as researchers never know the true functional form, this approach seemed compelling enough to be chosen. The problem becomes even more complex as the number of possible functional forms is large. Indeed any parametric estimation is by design incorrect, biases may increase if a wrong functional form is chosen. In this paper we have tested a rather extreme situation where we have used a very flexible form of the production function. Choosing a cubic specification of the production allows for a great local flexibility, but in the context of economic optimization only a concave function permits to choose inputs level that maximize profit, for this reason we impose monotonicity and concavity. On the other hand, a non-parametric simulation model enables to maintain the underlying economic properties such as monotonicity and concavity while at the same not imposing any arbitrary choice of a functional form.

The main contribution of the paper is the fact that it is one of the few attempts in policy modelling to test or validate methods. Indeed, validation of policy models is very complex because there is no possibility to do a real world experimental design. In addition, the DGP that are described are never truly known. Therefore, this paper attempts to compare methods based on simulations with different known DGPs. The findings from this validation attempt are a real eye-opener. In general Inverse DEA under CRS and the RLR model performs very poorly given the fact that we have generated a panel dataset of 1000 observations with different prices and free of any measurement or sampling bias. This amount of data is far beyond the real world policy simulation where often not more than 20 observations per farm are available and the amount of price variation is limited. In addition, in real world situations the DGP of a farm is likely to change during 20 years.

Further research could be conducted at a DGP at a higher level analysis such as farm level. Farms exhibit more variation in their technology and may change behaviour more gradually. Such model would also predict structural changes. Besides, an adjustment of the model to correct for bias and improve its efficiency will add value.

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